

UNIT - III

UNIT-III DIELECTRIC PROPERTIES

1) Dielectrics is an insulating material with high Specific Resistance.

The temperature coefficient of Resistance is negative.

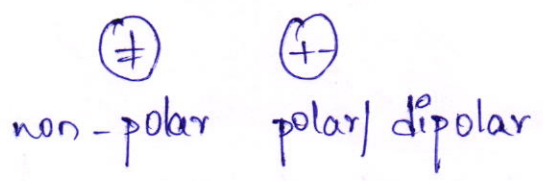
Dipole:- Slight/Small displacement of +ve and -ve charge is called dipole

Dipolement:- (M) Magnitude of Charge into the distance between the two charges.

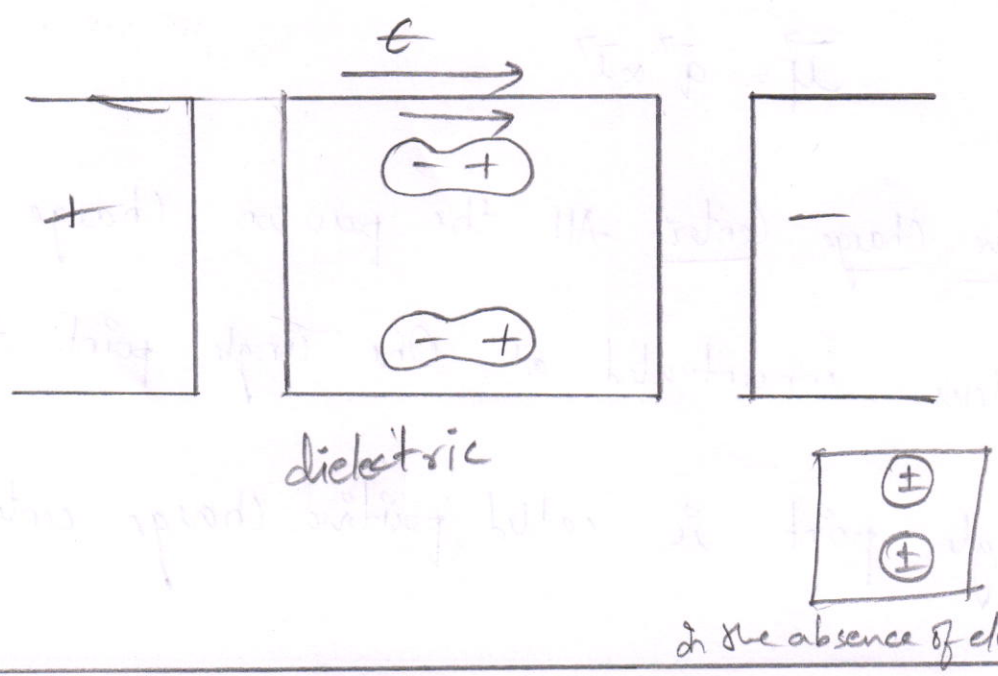
$$\vec{M} = q \vec{x}$$

Positive Charge Center:- All the proton charge of nucleus concentrated at One single point that single point is called positive charge center.

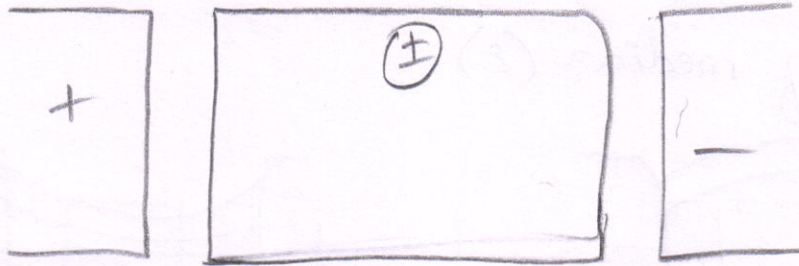
Negative Charge center :- The entire charge of the electrons in the atom concentrated at one single point which is called negative charge center.



Polarisation :- the process of producing the dipoles which are oriented along with field direction is called polarisation.



Polarizability :- (α) dipole moment for unit electric field is called 'α'



at $\epsilon = 0$
dipole moment for unit electric field is called α
the $\vec{u} = q \times \vec{l}$

at $\epsilon = 0 \Rightarrow \vec{u} = 0$ if $\theta = 180^\circ$

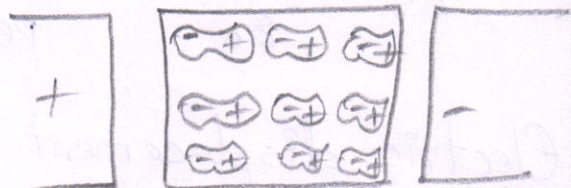
$\epsilon = 10 \Rightarrow \vec{u} = q(l)$

$$\therefore \vec{u} \propto \vec{E}$$

$$\vec{u} = \alpha \vec{E}$$

$$\alpha = \frac{\vec{u}}{\vec{E}} \text{ at } \epsilon = 1 \Rightarrow \alpha = \vec{u}$$

Polarization vector (\vec{P}) :-



\vec{P} = the magnitude of dipole moment for unit volume is called polarisation vector \vec{P} if it is the average magnitude of dipole moment

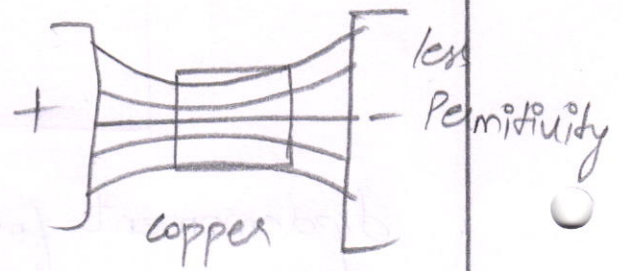
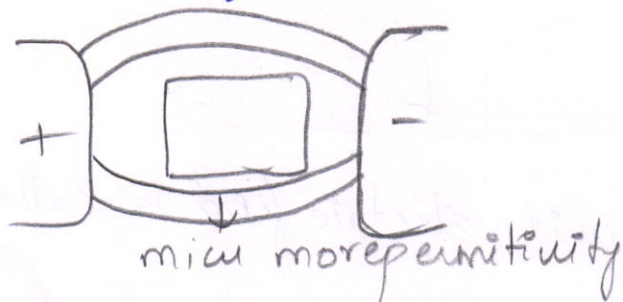
if N is the no. of atoms / molecules per unit volume

$$\vec{P} = N \vec{u}$$

Electric lines of force :-

the path of unit +ve charge in the electric field is called Electric line of force

Permittivity of media :- (ϵ)



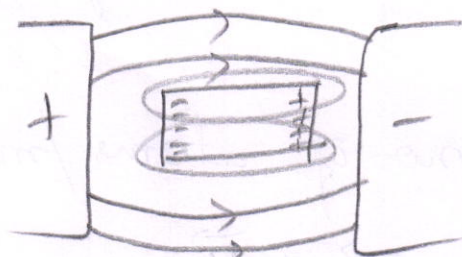
the materials ability to resist the electric lines of force (or) electric field is called permittivity of that material.

Relative permittivity of materials (or) dielectric constant

$$\epsilon_r = \frac{\epsilon}{\epsilon_0} = \frac{\text{Permittivity of material (media)}}{\text{permittivity of free space}}$$

Electric displacement vector P :-

the electric flux density is called Electric displacement



the displacement vector in the media is

$D =$ Electric lines of force due to External
Electric field + $\epsilon \cdot L$ of force due to polarised
charge

$$\therefore D = \epsilon_0 t + P$$

$$\epsilon_r \cdot \epsilon_0 t = \epsilon_0 t + P$$

$$P = \epsilon_0 t [\epsilon_r - 1]$$

$$D = \epsilon E \rightarrow \textcircled{2}$$

$$\vec{P} \propto E$$

$$\text{but } \epsilon_r = \frac{\epsilon}{\epsilon_0}$$

$$\vec{P} = \epsilon_0 \chi_e E$$

$$\Rightarrow \epsilon = \epsilon_r \epsilon_0$$

$$D = \epsilon_0 \epsilon_r E$$

How easily we can polarise the material we
need a physical quantity (susceptibility)

* If susceptibility is high we can easily polarise
the material

* If it is low then we cannot polarise the ma-
terial easily.

The relation b/w (ϵ_r & χ_e):-

we know $D = \epsilon_0 t + P$

But $P = \epsilon_0 \chi_e E$

$$\epsilon_0 \epsilon_r E = \epsilon_0 E + P$$

$$\epsilon_0 \epsilon_r E = \epsilon_0 E + \epsilon_0 \chi_e E$$

$$\epsilon_r = 1 + \chi_e$$

Various polarisation techniques.

1. Electronic polarisation
2. Ionic polarisation
3. Orientation polarisation
4. Space charge polarisation.

$$\text{charge density } \rho' \text{ of atom} = \frac{\text{charge}}{\text{Volume}} = \frac{-7e}{\frac{4}{3}\pi r^3}$$

$$\rho = \frac{-3}{4} \frac{-ze}{\pi R^3}$$

due to external electric field the force on electron cloud is

$$\text{Lorentz force } F_r = E(-ze) \quad \text{--- (2)}$$

The Coulomb force b/w two charge centres is

$$F_c = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

$$\Rightarrow F_c = \frac{1}{4\pi\epsilon_0} \frac{(ze) (\text{charge of the sphere of radius } r)}{r^2} \quad \text{--- (3)}$$

\therefore The charge on the sphere of radius r

= charge density \times volume of sphere of radius r

$$= \frac{-3}{4} \times \frac{ze}{\pi R^3} \times \frac{4}{3} \pi r^3$$

charge on the sphere of radius $x = -\frac{ze}{R^3} x^3$ (4)

Sub Eqn (4) in (3)

$$\therefore F_c = \frac{1}{4\pi\epsilon_0} (ze) \cdot \frac{(-ze)}{R^3} \times \frac{x^3}{x^2}$$

$$F_c = -\frac{z^2 e^2 x}{4\pi\epsilon_0 R^3} \quad (5)$$

at Equilibrium position $F_L = F_c$

$$(-ze) E = -\frac{z^2 e^2 x}{4\pi\epsilon_0 R^3}$$

$$\therefore x = \frac{4\pi\epsilon_0 R^3 E}{ze} \quad (6)$$

dipole moment $\vec{p} = (ze) \cdot x$

$$= \frac{(ze) \cdot 4\pi\epsilon_0 R^3 E}{ze}$$

$$\vec{p} = 4\pi\epsilon_0 R^3 E$$

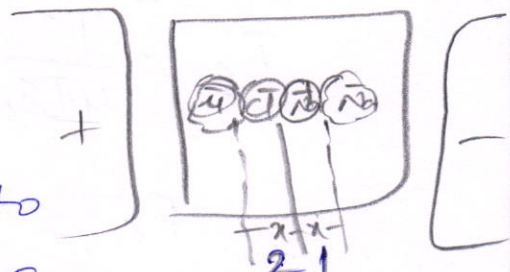
\therefore Electronic polarisability $\alpha_e = \frac{\vec{p}}{E}$

$$2) \alpha_e = 4\pi\epsilon_0 R^3$$

IONIC POLARISATION: The ionic polarisation is due to the displacement of cations & anions in opposite directions & occur in ionic solids. Suppose an electric field is applied in the x -axis direction. The +ve ion moves to the right

side by x_1 . -ve ion move towards left by x_2 .

In presence of external electric field the displacement of cation is x_1 , due to Lorentz force F_L cation is $F_L = eE$ — (1)



The restoring force of cation is $F_r = B_1 x_1$ — (2)

$$F_r = m\omega_0^2 \cdot x_1 \rightarrow (3)$$

here $B_1 = m\omega_0^2$ $B_1 \rightarrow$ restoring force const of cation

$m \rightarrow$ mass of cation

$\omega_0 \rightarrow$ angular frequency of molecule

$x_1 \rightarrow$ displacement of cation

at equilibrium state

$$F_L = F_r \Rightarrow eE = m\omega_0^2 \cdot x_1$$

$$x_1 = \frac{eE}{m\omega_0^2} \rightarrow (4) \text{ uly due to external electric field}$$

The Lorentz force on the Anion is $F_L = eE$ — (5)

The restoring force of Anion is $F_r = B_2 x_2$ — (6)

$$\text{but } B_2 = M\omega_0^2$$

$$(6) \Rightarrow F_r = M\omega_0^2 \cdot x_2 \text{ (7)}$$

here $M \rightarrow$ mass of anion - $\omega_0 \rightarrow$ angular freq. of molecule

$x_2 \rightarrow$ displacement of Anion

\therefore at equilibrium position $F_L = F_R$

$$eE = M\omega_0^2 \cdot x_2 \Rightarrow x_2 = \frac{eE}{M\omega_0^2} \quad \text{--- (8)}$$

\therefore Total distance b/w two ions is $x_1 + x_2$

$$= \frac{eE}{m\omega_0^2} + \frac{eE}{M\omega_0^2} \quad \text{--- (9)}$$

\therefore dipole moment $\vec{\mu} = e(x_1 + x_2)$

$$\vec{\mu} = e \left(\frac{eE}{m\omega_0^2} + \frac{eE}{M\omega_0^2} \right)$$

The ionic polarisability

$$\alpha_D = \frac{\mu}{E} = \frac{e^2}{\omega_0^2} \left[\frac{1}{m} + \frac{1}{M} \right]$$

\therefore polarisation vector $\vec{P} = N\alpha_D E$

$$\vec{P} = \frac{NEe^2}{\omega_0^2} \left[\frac{1}{m} + \frac{1}{M} \right]$$

ORIENTATION POLARISATION:-

When an electric field is applied on such molecules which possess permanent dipole moment, they tend to align themselves in the direction of applied field. The polarisation due to applied such alignment is called orientation polarisation.

The orientation polarisation greatly depends on temp

the orientation polarisability: $\alpha_0 = \frac{\mu^2}{3kT}$

$\mu =$ dipole moment of molecule

$k =$ Boltzmann constant

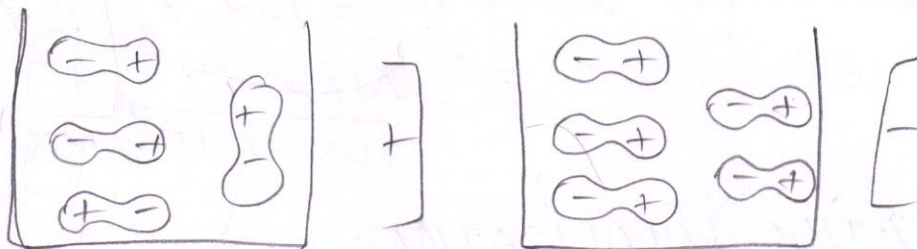
$T =$ temp of material

polarisation vector $P_0 = N\alpha_0 E$

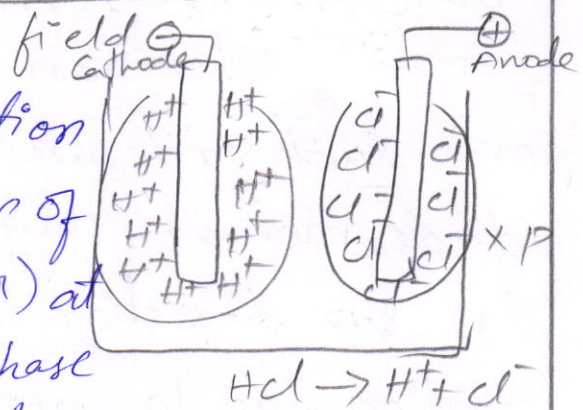
$$P_0 = NE \cdot \frac{\mu^2}{3kT}$$

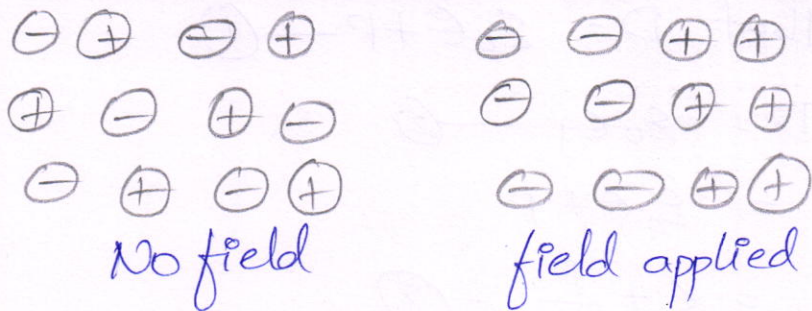
$$P_0 = \frac{N E \mu^2}{3kT_0}$$

Space charge polarisation: (Generally occurs in liquid)



Space charge polarisation occurs due to accumulation of charges at the electrodes (or) at the interface in a multiphase material as shown in fig.





the total polarizability in any material is

$$\alpha_T = \alpha_e + \alpha_p + \alpha_o + \alpha_s$$

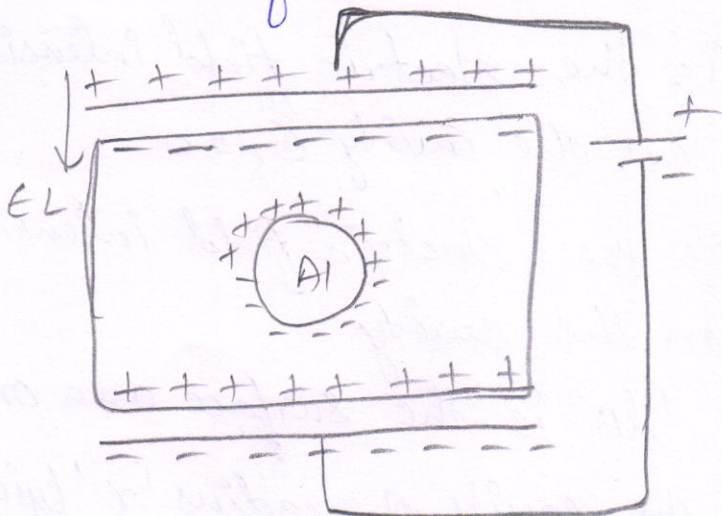
$$\alpha_T = 4\pi\epsilon_0 R^3 + \frac{e^2}{\omega_0^2} \left[\frac{1}{M} + \frac{1}{m} \right] + \frac{\mu^2}{3k_B} + 0$$

The polarisation vector $P = N \times E$

$$\therefore P = NE \left[4\pi\epsilon_0 R^3 + \frac{e^2}{\omega_0^2} \left(\frac{1}{m} + \frac{1}{M} \right) + \frac{\mu^2}{3k_B} \right]$$

This ϵ_0^n is called Langevin Debye Equation

* Internal fields in solids (Lorentz method) †



① Field $E_i = E$ is the electric field intensity at 'o' due to charges on the capacitor plates

we know that $D = \epsilon_0 E + P$ — (1)

at 'n' $D = \epsilon_0 E_1$ — (2)

$$\Rightarrow \epsilon_0 E_1 = \epsilon_0 E + P$$

$$E_1 = E + \frac{P}{\epsilon_0} \text{ — (3)}$$

② Field E_2 :- E_2 is the electric field intensity at 'A' due to polarised charges $E_2 = \frac{-P}{\epsilon_0}$

$$\therefore E_2 = \frac{-1}{4\pi\epsilon_0} \cdot \frac{q_1}{r^2}$$

$$= \frac{-q}{4\pi r^2} \cdot \frac{1}{\epsilon_0}$$

$$= \frac{-q}{A} \cdot \frac{1}{\epsilon_0}$$

$$E_2 = -P/\epsilon_0$$

$$P = \frac{q}{V}$$

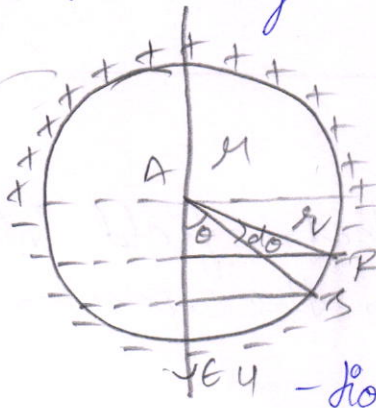
$$= \frac{q \times l}{V}$$

$$P = \frac{q \times l}{A \times l}$$

$$P = q/A$$

③ Field E_3 :- E_3 is the electric field intensity due to molecules in the cavity $E_3 = 0$

④ Field E_4 :- E_4 is the electric field intensity due to charge on the cavity



'dA' is the surface area on the cavity of radius 'r' lying b/w O & O + dO

'theta' is the angle b/w E_4 direction & AP

The surface area of loop on the cavity dA
 $= 2\pi (PA)(QR) \sin \theta$ — (6)

In $\triangle PAQ$ $\sin \theta = \frac{PA}{AQ} \Rightarrow PA = r \sin \theta$ — (7)

In $\triangle QAR$ $\sin \theta = \frac{QR}{AR} \Rightarrow QR = AR \sin \theta$
 $QR = r \sin \theta$ — (8)

Sub ϵ_0^n (7) & (8) in (6)

$$dA = 2\pi \times r \sin \theta \cdot r \sin \theta \cdot d\theta$$

$$dA = 2\pi r^2 \sin \theta d\theta$$
 — (9)

If dq is the charge on the dA area

$$dq = P \cos \theta \cdot dA$$
 — (10)

only normal component of polarization vector can give the charge

$$dq = P \cos \theta \cdot 2\pi r^2 \sin \theta d\theta$$
 — (11)

dE_A is the electric field intensity at A due to dq charge

$$dE_A = \frac{dq \cdot \cos \theta}{4\pi \epsilon_0 r^2}$$

$$F = EQ$$

$$E = \frac{1}{4\pi \epsilon_0} \times \frac{q}{r^2}$$

$$dE_A = \frac{P \cos \theta \cdot 2\pi r^2 \sin \theta d\theta \cdot \cos \theta}{4\pi \epsilon_0 r^2}$$

$$dE_A = \frac{P \cos^2 \theta \sin \theta d\theta}{2\epsilon_0}$$

the electric field intensity at 'A' due to entire

charge on cavity

$$\epsilon_u = \int_0^\pi d\epsilon_u = \int_0^\pi \frac{P}{2\epsilon_0} \cos^2\theta \cdot \sin\theta \cdot d\theta$$

put $\cos\theta = x$ say

$$-\sin\theta \cdot d\theta = dx$$

$$\Rightarrow \int_{-1}^1 \frac{-P}{2\epsilon_0} \cdot x^2 dx \Rightarrow \frac{+P}{2\epsilon_0} \int_{-1}^1 x^2 dx$$

$$\Rightarrow \frac{2P}{2\epsilon_0} \int_0^1 x^2 dx$$

$$\Rightarrow \frac{P}{\epsilon_0} \left(\frac{x^3}{3} \right)_0^1 \Rightarrow \frac{P}{3\epsilon_0} = \epsilon_u \quad \text{--- (13)}$$

∴ The total electric field intensity at 'r'

$$E = E_1 + E_2 + E_3 + E_4$$

$$= E + \frac{P}{\epsilon_0} - \frac{P}{\epsilon_0} + 0 + \frac{P}{3\epsilon_0}$$

$$E = E + \frac{P}{3\epsilon_0}$$

Internal
field

Internal field (a) Lorentz internal field E_r

CLAUSIUS - MOSOTTI EQUATION

Let us consider the external dielectric material. In that dielectrics & no permanent dipoles & ions. Due to that ionic polarisability & orientation polarisability is zero. The total polarisation $\alpha_T = \alpha_e + \alpha_I + \alpha_o$

$$\alpha_T = \alpha_e$$

The polarisation vector $\vec{P} = N\alpha_e \vec{E}$

$$\vec{P} = N\alpha_e \vec{E}_i \quad \vec{E}_i = \text{internal electric field}$$

$\alpha_e = \text{Electronic polarisability}$

$N = \text{No. of atoms (or) molecules per unit volume}$

$$P = N\alpha_e \left[E + \frac{P}{3\epsilon_0} \right]$$

$$\Rightarrow P = N\alpha_e E + \frac{N\alpha_e P}{3\epsilon_0}$$

$$\Rightarrow P \left[1 - \frac{N\alpha_e}{3\epsilon_0} \right] = N\alpha_e E$$

$$\Rightarrow P = \left[\frac{N\alpha_e E}{1 - \frac{N\alpha_e}{3\epsilon_0}} \right] \rightarrow (7)$$

We know that $D = \epsilon_0 E + P$

$$\text{but } D = \epsilon E \quad \Rightarrow D = \epsilon_0 \epsilon_r E$$

$$\epsilon_0 \epsilon_r E = \epsilon_0 E + P$$

$$P = \epsilon_0 \epsilon_r E - \epsilon_0 E$$

$$dE_{eq} = \frac{P \cos^2 \theta \sin \theta d\theta}{2 \epsilon_0}$$

the electric field intensity at 'A' due to entire charge on cavity

$$E_{eq} = \int_0^\pi dE_{eq} = \int_0^\pi \frac{P}{2 \epsilon_0} \cos^2 \theta \sin \theta d\theta$$

put $\cos \theta = x$ say

$$-\sin \theta \cdot d\theta = dx$$

$$\Rightarrow \int_{-1}^1 \frac{-P}{2 \epsilon_0} x^2 dx \quad \Rightarrow \quad + \frac{P}{2 \epsilon_0} \int_0^1 x^2 dx$$

$$\Rightarrow \frac{P}{2 \epsilon_0} \int_0^1 x^2 dx$$

$$P = \epsilon_0 \epsilon (\epsilon_r - 1) \quad \text{--- (2)}$$

from (1) & (2)

$$\epsilon_0 \epsilon (\epsilon_r - 1) = \frac{N k e \epsilon}{\left[1 - \frac{N k e}{3 \epsilon_0}\right]}$$

$$\left[1 - \frac{N k e}{3 \epsilon_0}\right] = \frac{N k e \epsilon}{\epsilon_0 (\epsilon_r - 1)}$$

$$\Rightarrow 1 = \frac{N k e}{\epsilon_0 (\epsilon_r - 1)} + \frac{N k e}{3 \epsilon_0}$$

$$\Rightarrow \left(\frac{\epsilon_r - 1}{\epsilon_r + 2} \right) = \frac{N \alpha_e}{3 \epsilon_0}$$

Clausius - Mosotti Eqⁿ ✓

Problem's: 1. Find the electric susceptibility of a dielectric gas having dielectric constant of 1.000041.

Sol: $\epsilon_r = 1 + \chi_e$

$$1.000041 = 1 + \chi_e.$$

$$\chi_e = 1.000041 - 1$$

$$= 0.000041$$

2. The radius of gas atom is 0.0622m calculate the polarizability of the gas & its relative permittivity. Given that the no. of atoms of the gas is 2.7×10^{25} atom/m³

PYRO ELECTRIC EFFECT:

Pyroelectric effect is the charge in spontaneous polarisation when the temp of the specimen is changed

The pyroelectric coefficient (λ) is defined change in polarisation for unit temperature

change of the specimen $(\lambda) = \frac{dP}{dt}$

Pyroelectric materials & their applications

1. pyroelectric materials such as Barium Titanate ($BaTiO_3$), $LiNiBO_3$ etc are used to make very good infrared detectors which can operate at room temperature

2. Materials such as PZT, $NaNbO_3$ & P_2T ceramics are used in the construction of Pyroelectric image tubes.

FERRO ELECTRICITY:-

The ferro electric property is the possibility of reversal/change of orientation of polarisation direction by an electric field. This leads to hysteresis in the polarisation P.

Ferro electric materials & their applications

Ferro electric materials & their properties

Ferroelectricity & pyroelectricity

PERRO e

32 crystal symmetry

12 are centro symmetry
(one does not show) any
property

20 are non-centro
symmetry
(Piezo electric
Property)

10 unique polar axis
(Pyro) electricity

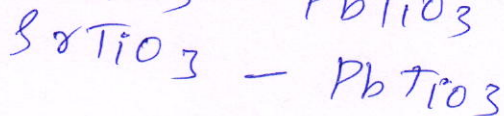
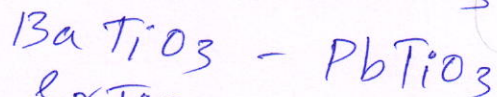
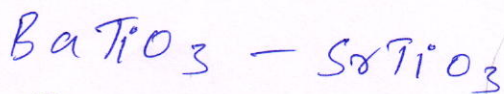
10 polarizations

ferroelectricity

Non ferro

Making use of piezo electric property, Ferro electric materials such as quartz, lithium niobate, barium titanate, lead zirconium titanate are used to make pressure transducers, ultrasonic transducers & microphones.

Ferroelectric semiconductors such as



are used to make

Resistors. resistors are used to measure of control the temp

Fig: Piezo, Pyro, Ferro electric materials are

active dielectric materials. A material can be piezoelectric, pyro. (or) Ferro electric only if its crystal symmetry. of the 32 crystal symmetry classes. 12 have centro symmetry remaining are asymmetric properties. All the materials in this 20 classes are piezoelectric materials if 10 of these - 20 classes have a unique polar axis - the existence of unique polar axis in crystal allows the appearance of the spontaneous electrical polarisation & hence these materials are pyroelectric. Few of the pyroelectric have the property of the Ferro electric

Thus all ferro electric are pyroelectric & piezoelectric. Still all pyroelectric are piezoelectric, but the converse is not true.